Notes and a Template on Queueing

Important Queueing Facts

Arrival Rate	λ	Arrival Time	$\frac{1}{\lambda}$
Service Rate	μ	Service Time	$\frac{1}{\mu}$
Utilization rate is $\rho = \frac{\lambda}{\mu}$			

Average Waiting Time (time spent in system, both waiting and being served)

$$W = \frac{1}{\mu - \lambda}$$

Average Waiting Time in Queuing Before Service

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Average Number of Customers in the System (waiting and being served)

$$L = \frac{\lambda}{\mu - \lambda}$$

Average Number of Customers in the Queue (average length of the line)

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Probability of Being in System (both waiting & being served) Longer than *t*

$$P(T > t) = 1 - P(T \leq t)$$
$$= e^{-(\mu - \lambda)t}$$

Note: t should be in the major unit of time. *i.e.*, 30 minutes = 0.5 hours

Probability That Number of Customers in System is Greater than *n* (waiting and being served)

$$P(N > n) = \rho^{n+1}$$

Identities:

$$L = \lambda W$$

$$L_q = \lambda W_q$$

$$W = W_q + \frac{1}{\mu}$$

A Template for Queuing: 1996 Midterm, Question #16

$U_1(0,1)$ Arrival Times	$U_2(0,1)$ Service Times
0.341	0.713
0.651	0.971
0.107	0.374
0.837	0.145
0.528	0.612

We are given the arrival rate of 12 and the service rate of 15. To be used in the formula, these must be converted to percentages. Given that the time interval in question in the hour, $\frac{\lambda}{60} = \frac{12}{60} = 0.20$ and $\frac{\mu}{60} = \frac{15}{60} = 0.25$.

Note: The negative exponential distribution (with parameter λ) is given by:

$$f(x) = 1 - e^{-\lambda x}$$

The inverse is:

$$\frac{-\ln(1 - f(x))}{\lambda}$$

Caution must be exercised with the denominator as λ represents the parameter of the exponential distribution, not the arrival rate. Both the arrival rate and the service rate will provide terms for the denominator here.

Build the simulation table:

u_1	Interarrival	Arrival	Waiting	u_2	Service	Exit	System Time
0.341	2.09	2.09*	0**	0.713	4.99	7.08	4.99
0.651	5.26	7.35***	0****	0.971	14.16	21.51*****	14.16
0.107	0.57	7.92	13.59	0.374	1.87	23.38	15.46
0.837	9.07	16.99	6.39	0.145	0.63	24.01	7.02
0.528	3.75	20.74	3.27	0.612	3.79	27.8	7.06
A	B	C	D	E	F	G	H
Given	$\frac{-\ln(1-A)}{\lambda/60}$		$\max\{G_{n-1} - C_n, 0\}$	Given	$\frac{-\ln(1-E)}{\mu/60}$	C+F(+D)	G-C

(*) By definition, the first arrival time is the first interarrival time.
(**) By definition, the first customer doesn't wait.
(***) $\text{Arrival}_{n-1} + \text{Interarrival}_n$
(****) $\max(\text{Exit}_{n-1} - \text{Arrival}_n, 0)$
(*****) $\text{Arrival}_n + \text{Wait}_n + \text{Service}_n$

IMPORTANT!!!

Don't forget to use λ in generating the first random number but use μ in generating the second random number!!!

Generally, the values of interest are the average waiting time (column D) and the average time in system (column H). Once the above table is constructed, it is simple to calculate the averages of the values contained therein.

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