

# CONCEPTS AND PROBLEMS IN CAPITAL BUDGETING

One of the most basic responsibilities of managers or, indeed, any economic agent, is to choose between various projects or investment opportunities such that the return is maximized or the cost is minimized. The difficulty here lies in determining the present value of some future event. This future event may have substantial uncertainty related to it. It may be contingent on other external factors. Many different variables must be considered in order to make an informed choice as to participation in a project. The situation becomes even more complicated when the choice is not whether to invest in a single project, but instead to choose among projects comparatively. In this brief note, we will consider some of the fundamental concepts necessary to understanding capital budgeting problems. In the end, you should be aware of a number of methods which are used and understand their strengths and weaknesses. There is no one definitive measure to judge projects by! Some methods will favor specific types of projects. This leaves an enormous amount of room for manipulation by the analyst. A thorough understanding of the processes and mathematics involved will help to prevent costly errors in the selection of projects.

## DISCOUNTED CASH FLOWS

Since all projects can generally be reduced to a stream<sup>1</sup> of positive and negative cash flows, one measure of a project's value is the "value" of those cash flows at the present time. If this is the case, we must ask ourselves how much some future sum of money is worth today. It is generally assumed that people prefer to have some amount of money immediately than to have that same amount of money at some point in the future. Thus, we know that *cash in the future is worth no more than cash today*. This implies that some form of discounting must be performed in order to compare the two.

One way to obtain such a value is to ask "what sum of money would I need to receive today to be indifferent to receiving \$1 in exactly one year?" That number, which must be less than \$1, would imply a discount rate.

$$\delta_1 x = 1 \tag{1}$$

A similar question could be asked for two years, and so on.

$$\begin{aligned} \delta_2 x &= 1 \\ \delta_3 x &= 1 \\ &\vdots \\ \delta_n x &= 1 \end{aligned} \tag{2}$$

The sequence  $\{\delta_i\}_{i=1}^n$  represents what is known as a **term structure** (a discount term structure in this case). We will deal more explicitly with this later. For the moment, however, let us consider what an appropriate value would be for  $\delta$ . If one thinks intuitively about this problem, one could imagine requiring, say, the rate of inflation to be taken into account. That is, we would want the real value of the cash flows to be received to remain constant or increase over time. If the future cash flow is uncertain, we may wish to consider a higher rate so as to include a **risk premium** for

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<sup>1</sup>In the case of uncertainty, they can be replaced by the expectations of the cash flow.

our investment. We may also wish to include a premium to compensate us for the “psychological pain” of waiting for the money.<sup>2</sup> For another example, assume we borrowed money to make the investment producing the future payoff. In this case, we may require that any adjustment in the future amount reflect our borrowing costs.

What should be becoming clear is that people generally target specific rates of return,  $r_i$ , which they expect for future cash flows. Thus, it is often more insightful to speak of returns rather than discount rates. In actuality, the two are related according to the following equality:

$$\delta_i = \frac{1}{1 + r_i} \quad [3]$$

In this case, the present value (PV) of some future cash flow (CF) is equal to the following:

$$PV(CF) = \frac{CF}{1 + r} \quad [4]$$

Although [4] explains the relationship, it omits several important factors such as the term of the investment producing the cash flow and exactly what the rate is. This leads us to ask a fundamental question in capital budgeting:

***What is the appropriate rate of return for discounting the cash flows?***

It so happens that a great deal of research in financial economics has largely solved this problem. While it is beyond the scope of this class to consider such research here, those who are interested should consult a corporate finance textbook (Brealey and Myers’ *Principles of Corporate Finance* is generally considered the classic). For our purposes here, we shall assume that the returns are given to us (as a result of a proper economic model, for example).

One concept that should be understood, however, is the potential for **arbitrage relationships** in the selection of a term structure of returns. Because of the desirability of basing such a structure on risk-free returns, U.S. Treasury bonds are frequently used. One can quickly see, therefore, that the rate used to discount some riskless cash flow must be the same rate as the equivalent term Treasury security. An arbitrage relationship is one in which a riskless profit can be made without the investment of any (net) capital. For this reason, the Treasury yield curve is often used to discount (riskless) cash flows; and, with an appropriate adjustment for risk (beta), to discount risky cash flows. This implies that the required returns for discounted cash flows change over time. In fact, it is likely that  $r_i \neq r_j, \forall i \neq j$ . So, when we are considering a series of cash flows, each may be discounted by a different rate. Often, in the problems you will encounter in this class, a constant rate of return will be used. If not, the present value of the stream of cash flows is given by [5], which is generally referred to as the DCF or Discounted Cash Flow model.

$$PV = \sum_{t=1}^T \frac{CF_t}{(1 + r_t)^t} \quad [5]$$

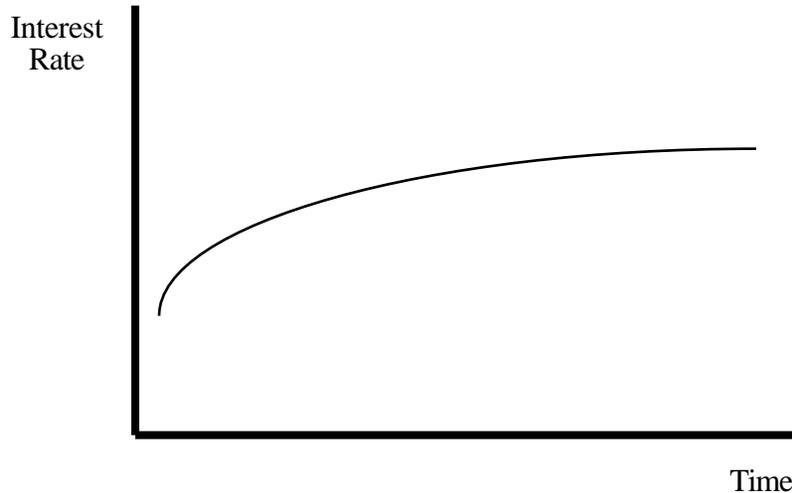
Here, each cash flow is discounted at its own rate. The series of rates of return represent what is

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<sup>2</sup>There appears to be some evidence (e.g., Prelec, D., and G. Loewenstein, *Anomalies in Intertemporal Choice*, *Quarterly Journal of Economics*, 1995) that, psychologically, people may derive pleasure from waiting. Prelec and Loewenstein term this “savoring”. Savoring is contingent upon the expectation of a pleasurable experience. We will ignore these claims in this analysis.

known as the **term structure of interest rates**, if the rates are the interest rates on riskless government bonds. Otherwise, they represent a term structure of returns. The term structures typically appear as in Figure 1.

**Figure 1**



With this structure, we can now consider evaluating investment opportunities. In the single investment case, the criterion for accepting an investment or project is that it have a positive **net present value**, or NPV. Because the NPV of a project includes all possible cash flows (both positive and negative) associated with a project, even a very small NPV would indicate acceptance. In comparing mutually exclusive projects, the appropriate measure is to consider the project with the highest NPV in the absence of other constraints. Any remaining funds should be committed to the project with the next highest NPV and so on.<sup>3</sup> In the next section, we will consider some potential problems with choosing among (mutually exclusive) projects based on NPV and some alternative approaches.

Finally, consider some possible refinements of the discounted cash flow model, [5]. Although we have not specified the period (it may be years, months, days, etc.), it may be useful to consider a form of [5] which takes into account a given compounding period,  $c$ . Here, we will presume that  $r$  represents an annualized return. In general, for finite compounding periods, the present value formula becomes [6].

$$PV = \sum_{t=1}^T \frac{CF_t}{\left(1 + \frac{r_t}{c}\right)^{tc}} \quad [6]$$

For example, for monthly compounding,  $c$  would be set to 12. It is easily shown that [6] can be reformulated to accommodate continuous compounding. Although it is often difficult to conceptualize the notion of truly continuous compounding, it is often used because using the exponential and logarithmic functions have desirable mathematical properties. Thus, we have [7].

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<sup>3</sup>Theoretically, all projects with positive NPVs should be accepted since the required returns for each project should take into account the increasing scarcity of capital. This presumes that firms and individuals do not face leverage constraints. As firms' borrowing increases, their required returns should also increase to reflect the additional risk of default contingent upon additional borrowing for the project in question.

$$\lim_{c \rightarrow \infty} \sum_{t=1}^T \frac{CF_t}{\left(1 + \frac{r_t}{c}\right)^{tc}} = \sum_{t=1}^T CF_t e^{-tr_t} \quad [7]$$

Some other formulas which are often used are those for annuities and perpetuities. Many of the situations you will encounter will even fall beyond these or involve combinations of various structures. For example, what is the value of an annuity which starts 5 years from now? The following formulas are all derived from the basic DCF model [5], however.

A **perpetuity** is a stream of constant cash flows which lasts forever. For example, in [5] or [6], if  $T$  were to become infinity, the expression would be a perpetuity and could be reduced to [8].

$$PV = \frac{CF}{r} \quad [8]$$

where  $r$  is a constant interest rate or, perhaps, the effective annual yield. If the perpetuity grows (a **growing perpetuity**) over time, the value can become even larger. In this case,  $g$ , the growth rate, cannot exceed  $r$ , the discount rate. In this case, the present value becomes infinite. Otherwise, the value is given by [8'].

$$PV' = \frac{CF}{r-g}, \quad \forall r > g \quad [8']$$

An **annuity** is a series of constant cash flows with a given time limit. For example, an annuity could be a stream of constant cash flows guaranteed for the next 10 years. The formula for a  $t$ -period annuity is derived from the difference between two perpetuities, one current, one in advance, and is given by [9].

$$PV = \frac{CF}{r} \left(1 - \frac{1}{(1+r)^t}\right) \quad [9]$$

## PROJECT SELECTION

We have seen how NPV and the DCF model are used to evaluate single project opportunities. What if, however, you are faced with a choice between several different projects, each with a different stream of cash flows? Let us be clear, at this point, at exactly what we mean by NPV. NPV includes both the costs and the revenues of a given project. Typically, the costs are realized immediately (time 0), and the revenues in future years. This need not be the case, however. Revenues may be constant, but costs may be experienced intermittently as machinery needs to be replaced, etc. The DCF model [5] can easily account for such events.

What we know about NPV tells us, then, that we should accept all positive NPV projects. Of course, there may well be more positive NPV projects than we have capital available for (see footnote 3). In this case, or if projects are mutually exclusive, we are forced to choose between positive NPV projects. As you will see, this is far from a simple problem. In very general terms, investors (whether individuals or corporations) face four primary sources of problems: differing terms, revenue/liability matching, funding policy constraints, and risk differences. In addition, the DCF model is not the only tool used for evaluating projects. Some of the other tools which are

used are the Internal Rate of Return (IRR) and payback period.<sup>4</sup> In this section, we will address each of these problems and tools in relation to NPV and the DCF model.

The problem of **differing terms** represents a weakness of using NPV as a selection criterion. For example, assume two competing projects, A and B. A has an NPV of \$100 and a term of 5 years. B has an NPV of \$110 and a term of 7 years. It would appear, on the surface, that B is the better project. However, A expires two years earlier than B. Thus, A frees up capital earlier than B which can be reinvested for additional profit.<sup>5</sup> The projects cannot effectively be compared because it may very well be the case that by freeing up the capital two years earlier, the firm could invest in a more profitable investment for the remaining two years. The situation is unclear. What is often done in these cases is to assume constant renewal. That is, to assume that each project can be renewed with the same conditions upon expiration of the term. Therefore, NPV can be used by simply considering the term to be the lowest common multiple of the two terms. While this works and will give you a number, it may not always be reasonable to assume constant renewal. Another criterion (particularly IRR or EUAC - Effective Uniform Annual Cash Flows) should be used.

**Revenue or liability matching** is often a significant consideration for start-up companies. They may wish to tie the receipt or payment of cash flows to related dates significant to their capital structure. For example, if a firm is to receive a large cash payment in two years, an investment opportunity which requires cash upfront or in one year may be rejected even if it has a larger NPV than one which doesn't require investment until the second year. The desire for matching cash flows is a real world constraint on the basic NPV models.

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<sup>4</sup>Additionally, there are the Effective Uniform Annual Cash Flows (EUAC), and several accounting based measures such as return on book value, flows-to-equity, and free cash flow. For those who are interested, the current "state-of-the-art" in corporate finance is the **real options** model. The real options model approach brings with it the mathematical sophistication of options pricing theory and has the ability to incorporate a broad spectrum of activities which cannot be considered with traditional methods. For example, the value of waiting or the value of *not* taking a project. An excellent reference in this area is Dixit, A., and R. Pindyck. *Investment Under Uncertainty*. (Princeton, NJ: Princeton University Press, 1994).

<sup>5</sup>That is, if A can easily earn more than \$10 in years 6 and 7, it becomes more profitable to accept A. The format used to state these returns is in **forward rates**. In this case, we would be interested in the two year rate of return five years from now or the *two year rate five years forward*. Such a rate is stated as  $r_{5,7}$ . Because the forward rate is determined by an arbitrage relationship, we can calculate exactly what the forward rate must be to maintain indifference (prevent arbitrage). This rate, derived below, represents a threshold value for project renewal. If one feels a project could be obtained at this rate in the fifth year, A becomes the preferred project. But still, the availability of such a project may not be known at the present time. Thus, NPV is still ill-suited to such analysis.

Let  $S$  index the "short-term" rate and  $L$  index the "long-term" rate (5 and 7 above). Through arbitrage, we can insure that the following relationship will hold:

$$\begin{aligned} [CF(1+r_s)^s](1+r_{s,L})^{L-s} &= CF(1+r_L)^L \\ (1+r_{s,L})^{L-s} &= \frac{(1+r_L)^L}{(1+r_s)^s} \\ 1+r_{s,L} &= \sqrt[L-s]{\frac{(1+r_L)^L}{(1+r_s)^s}} \\ \therefore r_{s,L} &= \sqrt[L-s]{\frac{(1+r_L)^L}{(1+r_s)^s}} - 1 \end{aligned}$$

A similar constraint is **funding policy**. Despite the existence of positive NPV opportunities, firms may have as standing policy a limit on the number or dollar amount of investments in particular areas. For example, a firm may wish to restrict the capital devoted to investments outside its core competency irrespective of the NPV of the project. A project manager at Microsoft may be aware of a great opportunity in the textile industry, but reject it because it fails to fulfill Microsoft's core competency.

Also, NPV does not consider the **riskiness of projects** as measured by the certainty of the cash flows. The incorporation of risk through adjustment of the required return is often insufficient. Although the values can be replaced by expectations and sensitivity analysis can and **should always** be done, using the DCF framework still does not adequately capture the full measure of risk in a project. Unfortunately, this problem is inherent in all standard measures of project selection. It is simply something which the decision maker must always be aware of.

Finally, although this note will not address them in detail, you should be aware of two popular alternate methods of project selection: payback and IRR. **Payback**, while popular, is a horrible method for evaluating choices between projects. It completely ignores the time value of money and ignores all cash flows after the payback period. Payback calculates for each project the time required before the sum of the cash flows exceeds the initial investment. The selection criterion is then to choose the project or projects with a payback period within a certain threshold. As a brief example of why this is such a poor measure, consider the following two cash flow streams:

A	-5	5	0	0	0	0	0
B	-5	1	2	2	2	2	20

The payback of A is 1 since the cost is recovered in the first year. The payback of B is 3 since the cost is not fully recovered until the third year. If, for example, the payback criterion set is 2, B would be rejected in favor of A even though the enormous payout at year 6 for B causes B's NPV to vastly exceed A's. Moreover, in real terms, A's NPV is negative!

**IRR**, as a measure, also has problems. The IRR can be thought of as the breakeven or required return on a project for it to have a positive NPV. The problem is that IRR often ceases to have meaning for projects whose cash flows alternate between positive and negative. It can also not be used to compare projects. As an example why not (it ignores the magnitude of the projects), consider the following two projects: A and B. A has an IRR of 45% and B has an IRR of 15%. Clearly - or so it would seem - A is a better project. However, the NPV of A is 10¢ while the NPV of B is \$1,000,000. Now which is a better project?

## CONCLUSION

As a decision maker, you must be aware of the strengths and weaknesses of various methods of evaluating investment projects. The key to doing this is achieving complete familiarity with the time value of money and the discounted cash flow model and its derivatives. Although any problem can be addressed with the DCF model, there are many instances when using the DCF model will result in faulty or substandard evaluation. By being able to recognize these cases and provide alternate measures of project evaluation, you can prevent costly mistakes from being made (such as losing your job).

Be aware of the fact that others either (1) may not know that using a particular measure may

result in an incorrect evaluation, or (2) may be fully aware and actively trying to deceive you for personal gain. In all cases, it is not so much the exact solution given by the model. More important is the ability to justify the assumptions made in reaching that solution. The key assumptions are the required rate of return, the variability and determination of the cash flows, and the comparative evaluation method. Questions you always need to be thinking about:

- How is the rate of return or discounting factor determined?
- How were the cash flows determined? (Taxes; Accounting profits or Real cash flows?)
- Why is this method being used rather than \_\_\_\_\_?
- What does using this method leave out?
- What would happen if \_\_\_\_\_ were used to evaluate this project?

It is simply good practice to evaluate a project from a number of different perspectives. If there are significant differences between the various evaluation techniques, extra care needs to be taken in selecting the appropriate project.

As a final question for you to think about, consider the following problem: You must decide which one of the following projects you will accept. You can only accept one. You are also required to justify why you selected that project. The relevant data are below.

	Time 0	Time 1	Time 2	Time 3	Time 4	Time 5	NPV	IRR	Payback	Var.
<b>A</b>	-10	5	5	2.5	2.5	0	2.26	21.8%	2	32
<b>B</b>	-10	-10	-5	20	20	20	17.88	31.7%	4	244
<b>C</b>	-5	0	0	5	5	10	11.98	39.9%	3	28

A has the best payback. B has the highest NPV. C has the highest IRR. And then there's the variance of the cash flows...what's a manager to do?

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