Review of Basic Geometry: Lines 4 Ways

[1] Equations of the Form $y = mx + b$

This is the basic equations of the line.

Slope: m
Intercept: b

[2] Equations of the Form $Ax + By = C$ (*e.g.*, $3x_1 + 2x_2 = 5$)

$$Ax + By = C$$
$$By = -Ax + C$$
$$y = -\frac{A}{B}x + \frac{C}{B}$$

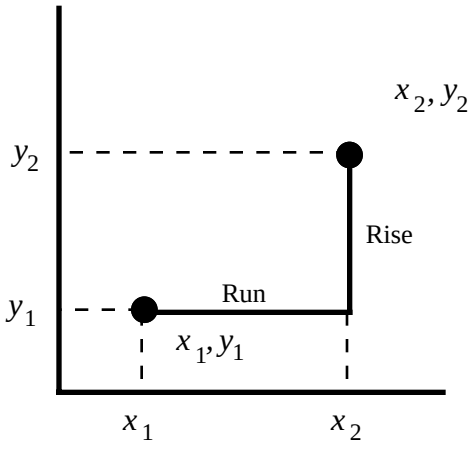
Therefore,

Slope: $-\frac{A}{B}$ ($-\frac{3}{2}$ in the example)
Intercept: $\frac{C}{B}$ ($\frac{5}{2}$ in the example)

[3] Equations from Two Points: $\{(x_1, y_y), (x_2, y_2)\}$

Any line through these two points must have a slope of:

$$= \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1}$$



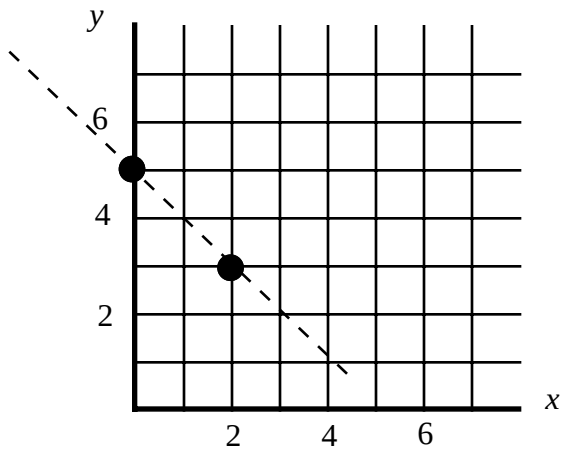
And the line must, obviously, include one of the points (we'll use the first one).

$$y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)x_1 + c$$

Therefore,

$$c = y_1 - \left(\frac{y_2 - y_1}{x_2 - x_1}\right) x_1$$

Just to check, suppose the two points are $\{(2, 3), (0, 5)\}$.



Slope = $\frac{5-3}{0-2} = \frac{2}{-2} = -1$

$$\begin{aligned} c &= 3 - (-1)(2) \\ &= 3 - (-2) \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

Therefore, the line is $y = -1x + 5$. Does this check? Is (0, 5) on the line?

5 = (-1) (0) + 5 √

[4] Equations from a Slope and a Point

The line must have a slope of *m* and pass through (x_1, y_1).

This should be easy now.

$$\begin{aligned} y &= mx + b \\ y_1 &= mx_1 + b \\ b &= y_1 - mx_1 \end{aligned}$$

Therefore, the equation of the line must be $y = mx + (y_1 - mx_1)$.