GRAD SCHOOL?

The problem with trying to solve time value of money (TVM) problems simply by memorizing formulas for zero-coupon discount securities and annuities is that very few actual situations are accurately characterized by doing so. Most applications of discounting involve relatively complex structures - even if they don't involve complex calculations. The true difficulty is not in working through the equations, but most often in figuring out exactly what is going on. The following is a good example: a simple problem with a complicated structure.

Recent undergraduate John isn't sure what his next step should be. Basically, he has two choices:

(1) Get a job now and don't go to graduate school
(2) Get a graduate degree and then get a higher paying job

At first glance it may seem like going to graduate school is always the best thing to do if it means making more money. But John, being a good decision theorist, realizes that he should carefully analyze this decision problem. Having taken Policy Analysis I, it is apparent that perhaps some kind of cost-benefit analysis is in order. Clearly, he should only go to graduate school if $V_G > V_J$ or, stated another way, if $V_G - V_J > 0$. That is, if the value derived from going to graduate school, V_G, includes a premium over taking a job immediately, V_J.

What are the costs? Of what value - *today* - are the benefits?

Part of being a good modeler is knowing when to abstract. Clearly, we can't possibly hope to model every element relevant to John's decision. What we do need to capture, however, are the important elements. And, because we are dealing with an abstracted case, it is imperative that we capture the elements we do wish to model as accurately as possible.

After examining his options, John concludes that based on his current education and experience he could get a job which paid $40,000 a year. His alternative is to spend $35,000 (tuition, housing, and living expenses) per year attending a two-year graduate program. Fortunately, to help offset this cost, John has $15,000 in savings. After graduation, John is confident that he'll make more than he could now, but he's not sure exactly how much more. Since he's uncertain about the effect this would have on his decision, the first step is to find the minimum post-graduate school salary that would make attending graduate school a better choice. If this isn't a reasonable figure (if it's far too high), then his decision is easy. Let's start with that.

We know the following variables:

$C_G = \$35,000$	The annual cost of the graduate program, paid at BOY
$S_J = \$40,000$	John's annual salary for a job today, paid at EOY
$S_G = ?$	Salary after graduate school, paid at EOY
$r_B = 10\%$	The rate at which John can borrow money
$r_m = 12\%$	The return on the stock market at which John can invest
$\pi = 3\%$	The annual rate of inflation
$W_0 = \$15,000$	John's current "wealth" (savings at time 0)

The first thing we can observe is that already a great deal of detail has been abstracted away. First of all, there are no taxes. Second, cash flows are experienced only at the beginning and end of the year. Finally, we're assuming that the various rates of return and interest do not change. Clearly, it would be possible to incorporate all of these elements in the model. However, as modelers we must always balance parsimony and accuracy. Is the predictive power gained by adding a stochastic interest rate sufficient to balance the added complexity of make such an addition? These are questions the modeler must seriously consider. What does the modeler know about these variables? How volatile are they? How directly do they affect the final outcome? If there is any question that they could *seriously* affect the final decision made, they should be incorporated. For this example, however, we will assume their effects to be minimal.

Our next step is to try and assign values to the one equation we need to know the answer to: $V_G - V_J > 0$. Let's start with the easy one first: what is the value of taking a job immediately? First, build a time line of John's cash flows:

	$t=0$	$t=1$	$t=2$	$t=3$	$\cdots$	$t=33$
Investment	15000	$15000 r_m$	$15000 r_m$	$15000 r_m$		$15000 r_m$
Salary	0	40000	$40000(1+r_\beta)$	$40000(1+r_\beta)^2$		$40000(1+r_\beta)^{32}$

Note that the salary starts at $t=1$ even though John starts work immediately ($t=0$) because we have assumed that he gets paid as a year end lump sum. If John took a job immediately, he could earn $40,000 a year for the rest of his life. Each of these years, on average, he expects that salary to increase by 5% for cost-of-living adjustments and performance bonuses. We will denote this rate of increase $r_\beta = 5\%$. In addition, he would be able to invest his savings of $15,000 and earn a profit on that. John is currently 22 and plans to retire at age 55. Therefore, we know that the value of choosing to work immediately is equal to the value of John's investments plus the value of his lifetime income stream.

$$V_J = \frac{15,000(1+r_m)^{33}}{(1+\pi)^{33}} + \sum_{t=1}^{33} \frac{40,000(1+r_\beta)^t}{(1+\pi)^t}$$

In words, this means that the value of taking a job immediately is equal to his current wealth, the profit that wealth will produce once invested, and the present value of 33 years of salary. Notice that both summations begin at $t=1$. This is because we are currently at $t=0$ and if John took a job immediately, he wouldn't be paid (per our assumptions) until the end of the year. Algebraically, we may express this as:

$$\begin{aligned} V_J &= \frac{W(1+r_m)^{33}}{(1+\pi)^{33}} + \sum_{t=1}^{33} \frac{S_J(1+r_\beta)^t}{(1+\pi)^t} \\ &= \frac{W(1+r_m)^{33}}{(1+\pi)^{33}} - \frac{S_J}{\pi - r_\beta}\left\{(1+r_\beta)\left[\left(\frac{1+r_\beta}{1+\pi}\right)^t - 1\right]\right\} \\ &= \$2,099,340 \end{aligned}$$

Therefore, the value of taking a job immediately and *not* going to graduate school is $2,099,340 using the assumptions made above. While we would also like to know how variable this value is to changes in the parameters, we will now turn our attention to assessing V_G.

In some sense, this is a trickier task. This is because this decision presupposes another decision: what should be done with John's current savings. If he goes to graduate school, should he use the savings to pay for part of the schooling or invest them and borrow the money. Or, should he invest and borrow and if so, at what times and in what amounts? Although each of these alternatives could be modeled separately, it is much easier to note that the rate of return on invested assets exceeds the rate charged for borrowing ($r_m > r_B$). And, since John faces no constraints on the amount he can borrow, the best decision is clearly to borrow the money to pay for school and invest the $15,000 at the higher rate. We'll assume that the loan to be used has a term of 10 years that John will make constant payments of principal and interest.

Assessing the value of the loan and the loan payments is in itself a TVM problem. Let's tackle this easy one first. We know John has to make equal payments (*P*) every year for the term of the loan (*L*). Since the bank will require the PV of the loan to be equal to the sum of the payments, we know that:

$$L = \sum_{n=1}^{N} \frac{P}{(1+r_B)^n}$$

$$\sum_{n=1}^{N} \frac{P}{(1+r_B)^n} - L = 0$$

Now we need to solve for *P*. Fortunately, the sum is convergent:

$$\sum_{n=1}^{N} \frac{1}{(1+r_B)^n} = \frac{1}{r_B}\left[\frac{(1+r_B)^T - 1}{(1+r_B)^T}\right]$$

From here, we can solve for *P*.

$$P = \frac{L r_B (1+r_B)^N}{(1+r_B)^N - 1}$$

If we assume that John borrows $35,000 at 10% for 10 years, his *annual* payments will be $5,696.09. For the purposes of incorporating this into our model, we'll assume for simplicity that John's loan payments are deferred (they're government subsidized) for the two years he's in school and that payments begin together at the end of his first year of work. Let's look at John's time table of cash flows:

	$t=0$	$t=1$	$t=2$	$t=3$	⋯	$t=33$
School	-35000	$-35000(1+\pi)$	0	0		0
Investments	15000	$15000r_m$	$15000r_m$	$15000r_m$		$15000r_m$
Salary	0	0	0	S_G		$S_G(1+r_\beta)^{29}$
Loan	35000	$35000(1+\pi)$	$-(2P+P\pi)$	$-(2P+P\pi)$		0

Now, let's add all of these elements together and determine the value of attending graduate school:

Let $N = 33$.

$$V_G = V_{\text{school}} + V_{\text{investment}} + V_{\text{salary}} + V_{\text{loan}}$$

Define P as the payment amount of the loan taken out to pay for graduate school:

$$P_{10} = \frac{C_G r_B (1 + r_B)^{10}}{(1 + r_B)^{10} - 1}$$

$$\begin{aligned} V_{\text{school}} &= -\sum_{t=0}^{1} \frac{C_G (1+\pi)^t}{(1+\pi)^t} \\ &= -2C_G \\ &= -\$70,000 \end{aligned}$$

$$\begin{aligned} V_{\text{investment}} &= W(1 + r_m)^N \\ &= 15,000(1.12)^{33} \\ &= \$631,373 \end{aligned}$$

$$\begin{aligned} V_{salary} &= \frac{1}{(1+\pi)^2} \left[\sum_{n=1}^{N-2} \frac{S_G (1 + r_\beta)^{n-1}}{(1+\pi)^n} \right] \\ &= -\frac{S_G}{(1+\pi)^2} \left[\frac{\pi^2 \left(\frac{1+r_\beta}{1+\pi}\right)^N + 2\pi \left(\frac{1+r_\beta}{1+\pi}\right)^N + \left(\frac{1+r_\beta}{1+\pi}\right)^N - r_\beta^2 - 2r_\beta - 1}{(1+r_\beta)^2 (\pi - r_\beta)} \right] \\ &= 38.4182 S_G \end{aligned}$$

$$\begin{aligned} V_{\text{loan}} &= -V_{\text{school}} - \sum_{t=3}^{13} \frac{2P + P\pi}{(1+\pi)^t} \\ &= 2C_G - 17.7046P \\ &= 2C_G - 100,847 \\ &= -\$30,847.10 \end{aligned}$$

Combining these together gives us the value of attending graduate school:

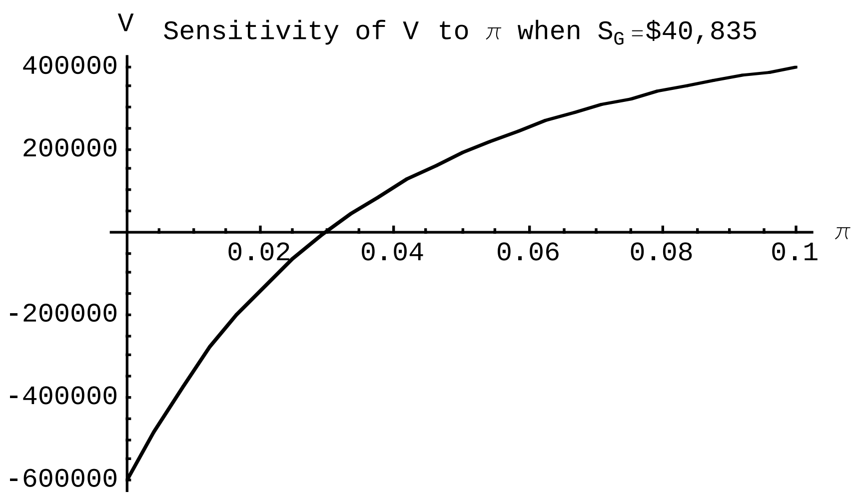
$$V_G = 530,526 - 38.4182 S_G$$

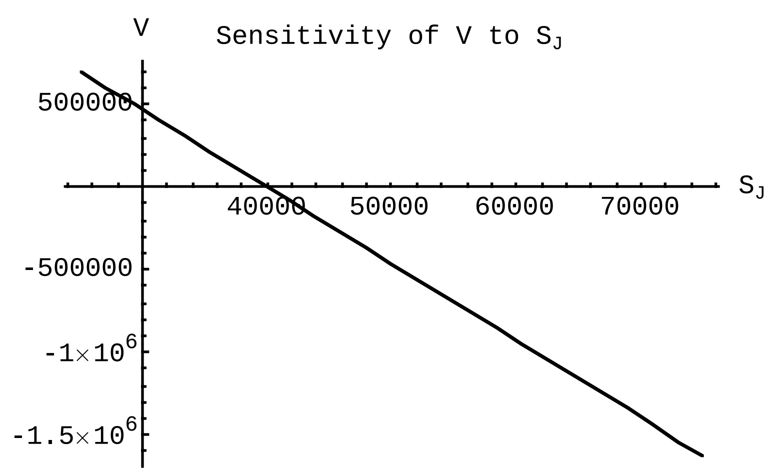
That said, we are interested in the *additional* value of attending graduate school. That is given by the following equation:

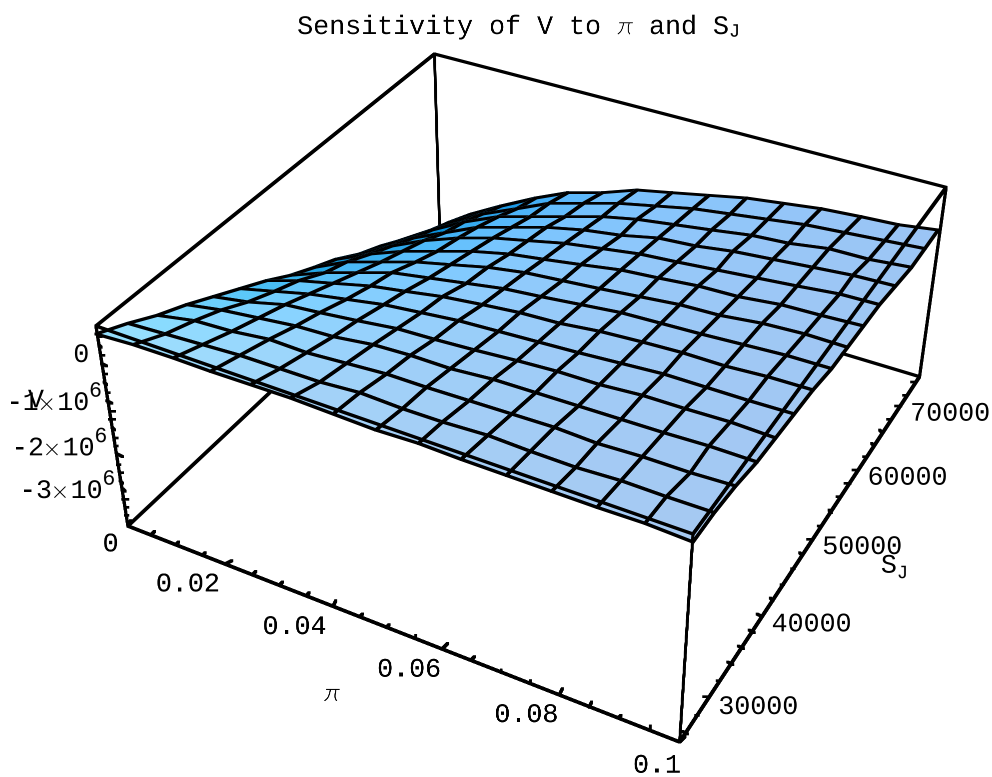
$$\begin{aligned} V_G - V_J &= 530,526 + 38.4182 S_G - 2,099,340 \\ &= 38.4182 S_G - 1,568,820 \end{aligned}$$

John's decision criterion was that if this amount was greater than zero, there was value in attending graduate school and he would attend school for another two years and then get a job. For this to be the case, $S_G > \$40{,}835.30$. Since this is approximately what he would make without a graduate degree, he felt it was safe to assume he could easily make more than this with a graduate degree and decided to attend graduate school.

Although John was comfortable with this decision, we may also ask how sensitive this decision is to changes in the parameters. In particular, we may want to know how much this value would change given changes in his current salary and the inflation rate. The easiest way to do this is to plot the objective function $(V_G - V_J)$ for different values of the parameters to assess inflection points.







David Rode
Decision Analysis and Decision Support Systems
Department of Social and Decision Sciences
Carnegie Mellon University
February 15, 1998